



Evolutionary game theoretical model for stable femtocells' clusters formation in HetNets

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ABSTRACT

Femtocell deployment is one of the key solutions to achieve the high data rate of the fifth generation mobile communication. Nevertheless, dense femtocell networks face several challenging tasks such as interference control and resource management. In this paper, we address the problem of resource allocation for heterogeneous networks (HetNets), namely dense femtocell networks, by forming stable clusters using an evolutionary game where femtocells learn from the environment and make their decisions considering the achieved payoff. In the literature, clustering has been proposed to organize network topologies by joining nodes (e.g. femtocells) with similar behaviors into logical groups. We focus on cluster stability that is important to obtain good network performance but can be difficult to achieve especially in ultra-dense and heterogeneous networks. In order to guarantee the cluster stability, we use the replicator dynamics that find the evolutionary equilibrium of the evolutionary game. Thus, by guaranteeing cluster stability the network performance is improved and the computational complexity is reduced. In addition, Particle Swarm Optimization (PSO) is used for the resource allocation algorithm that runs locally within each cluster owing to the fact that PSO has been proved to find a satisfying near-optimal solution while having the advantage of speeding up the optimization process. We run simulations for non-dense and dense femtocell networks taking into account two scenarios: fixed public users and public users that keep mobility such as pedestrians or cyclists. Simulation results show that the proposed solution is able to enhance the network throughput, to provide higher subscribers satisfaction, and to reduce the co-tier interference in dense femtocell networks.

1. Introduction

Fifth generation mobile communication (5G) has three targets to achieve: high data rates, low latency, and wide connectivity. These targets can be addressed by key technologies, such as heterogeneous networks (HetNets), massive multiple-input multiple-output (MIMO), and millimeter wave (mmWave) techniques [1]. HetNets, that comprise macrocells and small cells such as femtocells, are a cost-effective solution to tackle the increasing demand for network capacity. However, with increasing number of mobile users with random velocities in HetNets, attention should be drawn to the fact that the users will tend to move from one base station to another more frequently. In addition, the resources in each tier should be properly allocated considering that the number of femtocells will eventually increase.

The deployment of femtocells (FCs) in the macrocell coverage area is a promising and efficient solution owing to the fact that frequency bands can be reused between the macrocell and the femtocells. Furthermore, femtocells increase the coverage area in dead zones of indoor

environments, consume less energy than macro base stations (MBSs), and improve the system capacity when the number of femtocells increases. However, it should be noticed that femtocells are mostly deployed by end users without prior planning, which generates interference among femtocells also known as co-tier interference. The co-tier interference can be increased dramatically if the resources are not adequately managed within neighboring femtocells. Another type of interference, produced between tiers, is known as the cross-tier interference, where femtocell subscribers can be interfered by public users (PUs), that are not subscribed to nearby femtocells because they are unauthorized to connect to these femtocell or because these femtocells have no available capacity. The public users can also be interfered by nearby femtocells in the downlink communication.

Femtocell access control mechanisms are used to determine if public users are authorized to access this femtocell or not. The access control categories are: closed, open, and hybrid [2]. In the closed access case, the public users cannot access the nearby femtocells but they can

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generate interference that affects the downlink communication of the subscribers. The open access category allows any user to benefit from FCs' services. This approach requires tight coordination between FCs and their macrocell that may result in traffic congestion over the backhaul connections. In the hybrid access case, a public user can access a nearby FC but some capacity of this FC is reserved for its FC's subscribers. This approach can combine the benefits of the two previous access control categories and overcome their limitations. Due to this potential, in this paper we consider the hybrid access control.

In this paper we address these issues by proposing a resource allocation solution on femtocell clustering that uses an evolutionary game enabling femtocells to switch clusters to obtain a higher payoff. This evolutionary game considers a scenario with dense-femtocell deployment and a random walk mobility model. Initially, a set of femtocell clusters is formed using the K-means algorithm. Then, public users are allocated to nearby femtocells depending on their demanded data rate. This allows to determine the payoff of every cluster and consequently the average payoff of all clusters. At this point, our evolutionary game determines the set of stable clusters by using the replicator dynamic, i.e. set of clusters with a payoff similar to the average payoff. Finally, a distributed resource allocation algorithm will run locally within every cluster using the Particle Swarm Optimization (PSO) technique.

In our previous work [3], a stable cluster formation of femtocells was proposed based on a coalitional game and the e-core concept stability criteria where neither mobility and dense femtocell deployment were considered. Thus, the main goal of this work is to maximize the throughput of the femto-tier by means of a cluster based resource allocation approach for dense femtocell networks using an evolutionary game to form stable clusters. To the best of our knowledge, the majority of the previous cluster based resource allocation approaches do not guarantee the stability of the clusters in dense femtocell networks. The main contributions of this work are:

- Application of an evolutionary game to form femtocells clusters that reduces the complexity of resource allocation in dense-femtocell networks, in such a way that the resource allocation algorithm based on the PSO technique can run locally within each cluster.
- Use of the replicator dynamic of the evolutionary game theory to guarantee the clusters' stability and to avoid the reallocation of resources due to the constant changes in the cluster configuration.
- Analysis of the system performance when a mobility model is considered for public users in dense-femtocell networks.

The rest of this paper is organized as follows. Section 2 presents a motivational scenario. Section 3 describes the related work. The system model, problem formulation, and user mobility are presented in Section 4. Section 5 details the fundamentals concepts of the evolutionary game theory, the replicator dynamic, and the stability concept. The main components of the proposed model for clustering and resource allocation, and the benchmark models are explained in Section 6. The simulation results are discussed in Section 7. Finally, Section 8 concludes the work.

2. Motivational scenario

In game theory, it is often assumed that all players have knowledge of all network information at every moment. In this case, each user has to exchange large amounts of information, which makes game theory not suitable for large-scale networks. To cope with this issue, one can apply the evolutionary game theory where it is assumed that players have bounded rationality, which reduced the complexity and makes it suitable for densely-deployed femtocell networks. These characteristics allow players to adapt their strategy to obtain a higher payoff by replication. In addition, strategies that are more fruitful dominate over time which leads to evolutionary equilibrium.

Table 1
Payoff per cluster for 10 femtocells using the LBC model.

Iterations	Payoff per cluster c					Avg. payoff
	c_1	c_2	c_3	c_4	c_5	
1	2.09	1.02	2.22	1.01	1.99	1.88
2	1.99	2.37	2.58	1.11	2.03	2.13
3	2.89	2.62	1.70	1.41	1.57	1.97
4	2.38	2.27	1.81	0.96	2.11	1.97
5	2.83	2.37	1.87	1.21	2.14	2.13
6	3.06	2.47	2.32	1.16	1.72	2.18
7	2.23	2.27	1.93	1.06	2.14	1.99
8	2.39	2.57	1.85	1.26	2.21	2.08
9	2.47	2.47	2.13	1.41	1.58	1.99
10	2.44	2.52	1.54	1.16	2.27	1.99

Motivated by the characteristics of EGT presented in the previous paragraph, we propose a model that provides stable clusters by means of the replicator dynamic. This is the main contribution of this work that also constitutes the main difference when compared to the Load-Balanced Clustering (LBC) model [4], which is used as a benchmark model. The LBC model proposed the grouping of femtocells into clusters of similar size with no guarantee of stability as can be seen in the example illustrated in Table 1. The table shows the payoff per cluster and the average payoff of all clusters obtained with the LBC model. According to the replicator dynamics, the stability is achieved when all formed clusters tend to have payoffs equal to the average payoff of all clusters [5] but Table 1 shows that the LBC model does not reach this condition. In particular the payoff of the fourth cluster is always below the average payoff (see the highlighted column in Table 1). In contrast, our model uses the replicator dynamic as the stability criteria that is explained in Section 5.2.

3. Related work

To solve the resource allocation in a macro-femtocell network, several approaches that work with clustering techniques have been proposed. In [6], the clustering is performed based on femtocells positions. Specifically, the K-means algorithm executes an iterative data-partitioning algorithm based on a given cluster size and cluster number. Then, the resource allocation takes into account QoS requirements and cross-tier interference. In [7], interfering femtocells are grouped into clusters while the subchannel allocation is performed by a cluster head, the femtocell with the highest degree of interfering neighbors. In [8], the channel allocation problem is performed by using the cluster topology for high density networks. In this case, based on the K-means algorithm, femtocells are divided into different clusters that can self-adapt to a dynamic network topology.

Recently, the resource allocation problem has been tackled by game theory and evolutionary game theory models. In [9], a robust Stackelberg game that aims to achieve robust equilibrium is proposed for the resource allocation considering the macro-base stations demanded capacity. In [10] a centralized user-centric merge-and-split rule based coalition formation game is proposed to estimate interuser interference. Interference management based on hierarchically joint user scheduling and power control is proposed in [11] to alleviate co-channel interference. Further, a Stackelberg game is formulated between macro base station and femtocell base stations in order to determine the optimal transmission power.

An evolutionary game is proposed to develop an energy efficient subcarrier allocation method. The authors consider the height of base station's antenna and secondary users, the total data transmission rate limit, total power consumption constraint and power consumption constraint on a single carrier [12]. In [13], EGT is applied to cell

selection in two-tier femtocell networks with different access methods and coverage area. In [14], a centralized evolutionary game theoretic framework is proposed to form balanced femtocell clusters based on a distributed power control, a bankruptcy channel allocation, and an evolutionary clustering. In [15], a novel threshold pricing scheme is presented for offloading macro users to small cells. Based on an evolutionary game model, the behavioral dynamics of the macro users under two pricing strategies is analyzed. Distributed resource allocation is addressed with EGT in [16]. The authors proposed two game models based on the achievable signal-to-interference-plus-noise ratio (SINR) and data rate. In [17], BS allocation problem is modeled as an evolutionary game with QoS guarantee. In addition, a distributed learning-based algorithm is proposed to demonstrate the convergence to the evolutionary equilibrium.

The main limitations of the prior related work can be summarized as follows:

1. Most of the previous approaches propose a solution for femtocells working in closed access mode [6,18]. This is not suitable for public users that are nearby the coverage area of femtocells since the access to these femtocells is not granted. Thus, public users will try to connect to the MBS resulting in a large increase of the cross-tier interference.
2. Lack of femtocell cluster formation algorithms that guarantee the stability of the clusters. Cluster stability is a very important issue since it prevents the femtocells from abruptly changing from existing cluster to another one, which would lead to an unstable network.
3. Not taking into account the mobility of public users in the resource allocation approaches for dense femtocell networks. In a dense-femtocell network, the number of public users changing from one femtocell to another or from a femtocell to a macrocell or vice-versa increases with the users' mobility and this can cause instability in the network.

To overcome the above limitations, we propose a distributed resource allocation framework that maximizes the femto-tier throughput while enhancing the satisfaction of femtocell subscribers. The proposed solution focuses on the clustering of femtocells based on an evolutionary game model where stability is achieved by using the replicator dynamic. The main reason for using EGT is the reduced amount of information that would be exchanged among femtocells which makes it suitable for dense femtocell networks.

We previously addressed the resource allocation in macro-femtocell networks in [19] using an equal distribution of the resources among femtocells within a cluster. However, this method does not guarantee the same subscriber satisfaction for the cooperative femtocells. Our latest work, [3], tackled the resource allocation issue with a coalitional game that groups femtocells into clusters. In addition, Shapley value was used to guarantee the fairness distribution of resources and stability was demonstrated by means of the e-core concept. The main differences between the current work and our prior works, [3,19], are the evolutionary game used to group femtocells into clusters and the stability criteria based on the replicator dynamic. In addition, the present work analyzes scenarios with and without mobility for public users by means of a mobility model to assign random speed to the public users. It is also worth noting that our proposal evaluates the system performance in a dense femtocell network.

4. System model

We consider the downlink transmission of an OFDMA macro-femtocell network with several femtocells, FCs, deployed under the coverage area of a macrocell, MC, as illustrated in Fig. 1. Let $F = \{F_1, F_2, \dots, F_f, \dots, N_F\}$ be the set of femtocells and $|F| = N_F$. The set of available subcarriers is denoted as $SC = \{S_1, S_2, \dots, S_s, \dots, N_S\}$ and B_s denotes the bandwidth of each subcarrier. In order to eliminate the

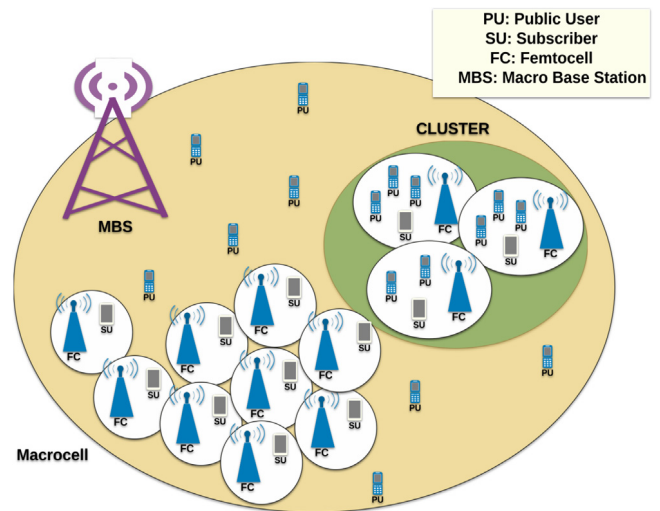


Fig. 1. Example of a dense macro-femtocell network with 3 femtocells forming a cluster and 9 stand-alone femtocells. SU and PU represent the subscriber and public user, respectively.

cross-tier interference, SC is partitioned into two disjoint sets, SC_{macro} and SC_{femto} , in such a way that their intersection is the empty set and their union is SC . These two disjoint sets represent the set of subcarriers for the macro-tier and the femto-tier, respectively.

For convenience, in this paper we assume that each femtocell can grant service to one subscriber so the femtocell obtains more resources from the macrocell than in the case of FCs having multiple subscribers. However, it should be underlined that our approach is still valid for the cases with more than one subscriber per femtocell. It is assumed also that femtocells use the hybrid access mode allowing them to grant service to nearby public users. The demanded data rate for subscribers and public users is randomly generated.

4.1. Problem formulation

In this work, we focus on the resource allocation for subscribers and public users served by femtocells within a cluster. In the proposed scenario, femtocells join a cluster through a clustering algorithm based on evolutionary game theory. A set of subcarriers are allocated to each cluster and they can be reused among different clusters. The set of clusters and mobile users are represented by $C = \{C_1, C_2, \dots, C_c, \dots, N_C\}$ and $MU = \{U_1, U_2, \dots, U_u, \dots, N_U\}$, respectively. In addition, the set of femtocells within a cluster c is denoted as F^c .

The SINR at mobile user u being served by femtocell f for the subcarrier s is given by

$$SINR_u^{f,s} = \frac{\alpha_u^f P_u^{f,s}}{PL_u^{f,s} \times (\sigma + \sum_{h \in (C \setminus c)} \sum_{f \in \{F^h\}} I_u^{f,s})}; \quad f \in F^c, u \in MU \quad (1)$$

where $P_u^{f,s}$ is the transmitted power from femtocell f to user u in subcarrier s , $PL_u^{f,s}$ is the path loss, $I_u^{f,s}$ represents the interference generated for users within clusters h , and σ is the noise power. In our model, the interference source for the femto-tier is the inter-cluster interference that is represented by the second term of the denominator in (1).

The propagation model used to estimate the SINR ratio is similar to the one presented in our previous work [20], and is given by:

$$PL_u^{f,s}(\text{dB}) = 10 \log_{10}(d_{uf}) + 37 \quad (2)$$

where d_{uf} is the distance (in meters) from user u to femtocell f in accordance with the carrier frequency used for femtocells [21]. We

verified that the simulation results presented in Section 7, that are based on Eq. (2), are also valid for the 3GPP path loss model [22] since while there is a slight difference in throughputs between the two path loss models, the relative differences between the performance metrics of the tested algorithms and models are practically identical for both path loss models.

The achievable data rate of mobile user u served by a femtocell f in subcarrier s is represented by

$$R_u^{f,s} = \alpha_u^f \cdot \beta_u^{f,s} \cdot B_s \cdot \log_2(1 + SINR_u^{f,s}) \quad (3)$$

where α and β are the binary variables that represent user base station association and subcarrier allocation per user, respectively. In other words, α_u^f determines if user u is served by femtocell f while $\beta_u^{f,s}$ indicates if subcarrier s is allocated to user u in femtocell f . In [23], the authors determined that the potential number of clusters is given by the Stirling number of the second kind (Bell number), which grows exponentially with the number of femtocells where the complexity is defined as $\mathcal{O}(f^f)$. To reduce the complexity, we decompose the maximization problem into two sub-problems: the clustering sub-problem that forms the femtocell groups and the resource allocation sub-problem that maximizes the throughput of each cluster. It is worth noting that our approach finds a satisfying near-to-optimal solution within each cluster through the use of the PSO algorithm that is used for the resource allocation.

The clustering sub-problem is solved by using an evolutionary game where the femtocells are considered as players of the game. In this game, the femtocells' allocation evolve towards balanced clusters with payoffs close to the average payoff using Algorithm 1. The goal of the clustering is to allocate resources within each cluster, to improve the femtocells' performance, and to reduce the inter-cluster interference. As result, the femtocells increase their subscriber's rate owing to the fact that they receive more subcarriers by granting access to nearby public users. Thus, the increase of the network' throughput is guaranteed by the increase of every cluster's throughput. In addition, our solution focuses on forming stable clusters. To accomplish this task, we use a stability criterion based on the replicator dynamic of the evolutionary game theory. Therefore, when stability is reached, the solution that maximizes the throughput of each cluster is equivalent to maximizing the sum of the throughputs of all clusters, since the clusters do not change constantly.

On the other hand, the resource allocation-subproblem, that considers the maximization of the throughput within a cluster c formed by the subset of femtocells F^c , is run by each cluster head using Algorithm 2. The cluster head is the femtocell with the highest number of neighboring femtocells and it is responsible for the resource allocation among all the members of the cluster.

The objective function is formulated as follows:

$$\begin{aligned} & \max_{\alpha, \beta, P} \sum_{u \in MU} \sum_{f \in F^c} \sum_{s \in SC} \alpha_u^f \cdot \beta_u^{f,s} \cdot B_s \cdot \log_2(1 + SINR_u^{f,s}) \\ & \text{subject to} \quad C1 : \sum_{f \in \{F^c\}} \sum_{s \in \{SC\}} \beta_u^{f,s} \leq 1; u \in MU, \\ & \quad C2 : \sum_{f \in \{F^c\}} \sum_{u \in \{MU\}} \sum_{s \in \{SC\}} \alpha_u^f \beta_u^{f,s} \leq \\ & \quad N_S - \sum_{u \in \{MU\}} \sum_{s \in \{SC\}} \alpha_u^{MC} \beta_u^{MC,s} \quad (4) \\ & \quad C3 : \log_2(1 + SINR_u^{f,s}) \geq \alpha_u^f \beta_u^{f,s} \gamma_f; \\ & \quad u \in MU, f \in \{F^c\}, s \in \{SC\}, \\ & \quad C4 : \sum_{f \in \{F^c\}} \alpha_u^f \leq 1; u \in MU, \\ & \quad C5 : B_s \times \sum_{s \in \{SC\}} \beta_u^{f,s} \gamma_f \geq \alpha_u^f \times D_u; u \in MU. \end{aligned}$$

Constraint C1 guarantees that a subcarrier being used in the macro-tier is not used by any cluster in the femto-tier. Constraint C2 represents the upper bound for the subcarriers allocated to the cluster c . Constraint C3

provides that the spectral efficiency achieved by a mobile user u within a cluster has to be higher or equal to a target spectral efficiency (γ_f). Constraint C4 guarantees that one user is assigned to only one base station. Constraint C5 defines the lower bound for minimum data rate for mobile users where D_u represents the requested data rate demand of mobile user u .

In order to reduce the resource allocation complexity for macro-femtocell networks, we propose to use cluster formation techniques. The optimal cluster configuration can be found by applying an exhaustive search. However, an exhaustive search would require long running times since the number of possible cluster configuration increases as the numbers of femtocells increase.

4.2. User mobility

The mobility of public users is modeled using the Random Walk Mobility model. Random Walk was proposed to mimic the movement behavior of mobile nodes which are considered to move in an unexpected way. It is a memoryless model where the information of the previous velocity and direction is not retained [24].

The main characteristics of Random Walk Mobility are summarized as follows:

- The speed and direction of the nodes are changed each time interval and it has zero pause time.
- Speed $v(t)$ is chosen from previously defined ranges $[V_{min}, V_{max}]$ by each node which follow a uniform distribution or Gaussian distribution.
- The direction $\theta(t)$ is also chosen by each node from the ranges $[0, 2\pi]$.
- Every movement is made either in constant time interval t or in constant distance traveled d .
- The node moves with the velocity vector $[v(t) \cos(\theta(t)), v(t) \sin(\theta(t))]$, during time t .

According to [25], the users velocities are classified as low (from 0 to 15 km/h), medium (from 15 to 30 km/h), and high (above 30 km/h). In the present work, we considered low velocities for the public users.

4.3. Model parameters

Table 2 presents the basic parameters used in the proposed model. The parameters are classified into three categories: system, input, and output parameters. The system parameters describe the network features. The users' requirements and locations are presented as the input parameters. The output parameters are the set of stable clusters and the bandwidth and power allocated to all users.

5. Evolutionary game theory fundamentals

Evolutionary game theory was proposed by John Maynard Smith who adapted the traditional game theory to the concept of evolution by natural selection. In brief, evolutionary game theory models the behavior of large populations of individuals with bounded rationality. In traditional game theory, the strategies are fixed while in evolutionary game theory strategies evolve. In our case, the populations of individuals corresponds to the population of femtocells. In particular, the femtocells observe the behavior of other femtocells and make decisions based on their payoff and the average payoff of all other femtocells. Therefore, femtocells will be tempted to choose those strategies that give better payoffs. In this manner, those strategies will predominate with time.

Table 2
Model Parameters of the k-EGT model.

System Parameters	
Symbol	Description
C	Set of clusters
SC	Set of available subcarriers
MU	Set of mobile users
F	Set of deployed femtocells
F^c	Set of FCs in cluster c
B_s	Bandwidth per subcarrier
BW_c	Bandwidth reserved for the clusters formation
N_f	Number of femtocells
N_C	Number of clusters
N_S	Number of subcarriers
P_f^{total}	Total transmitted power in femtocell f
r_{MC}, r_f	Radio in macrocell and femtocells
γ_s^f	Subcarrier s spectral efficiency in femtocell f
γ_f	Target subcarrier spectral efficiency in femtocell f
f_c	Carrier frequency adopted by the MC (in MHz)
σ	Average thermal noise power
$x_{f,c}$	Individual payoff of FC f in cluster c
$v(t)$	Users' velocity
I_{th}	Interference threshold
Input Parameters	
R_{SU}^f	Subscriber data rate demands in FC f
R_{PU}^f	PU data rate demands in FC f
D_u	Requested data rate demand of mobile user u
d_{uf}	Distance from mobile user u to FC f
α_u^{MC}	User u assigned to MC
$\beta_u^{MC,s}$	Subcarriers s allocated to user u in MC
Output Parameters	
α_u^f	User u is assigned to BS f
$\beta_u^{f,s}$	Subcarrier s allocated to user u in femtocell f
$P_u^{f,s}$	Transmitted power in DL transmission between femtocell f and user u
$R_u^{f,s}$	Data rate allocated to MU u served by femtocell f in subcarrier s

Definition 1 - Evolutionary game. An evolutionary game can be defined as $G = (F, S, \pi^f(S_k)_{f \in F, S_k \in S})$ where F is the set of players (femtocells in our case), which constitutes the population in an evolutionary game; S is the set of all strategies available to each player that is defined as $S = \{S_k\} = \{a_1, a_2\}$ where actions a_1 and a_2 refer to staying in the femtocell current cluster and to switching to another cluster, respectively, and $\pi^f(S_k)$ is the femtocell f payoff, at time $t + 1$, obtained by using strategy k at time t .

Payoff function. The payoff of femtocell f is defined as

$$\pi^f = \sum_{u \in MU} \sum_{s \in SC} \alpha_u^f R_u^{f,s} \quad (5)$$

where α_u^f is the binary variable that determines if user u is served by femtocell f and $R_u^{f,s}$ is the allocated data rate of mobile user u served by a femtocell f in subcarrier s .

The main goal of a femtocell is to maximize its throughput represented as payoff, Eq. (5). Thus, evolutionary game theory allows a femtocell to leave the current cluster and choose another cluster that increases its payoff. As a result, the femtocell cluster allocations evolve to balanced clusters where femtocells tend to have payoffs equal to the average payoff of the whole population. The average payoff of all clusters is defined as

$$\bar{\pi} = \frac{\sum_{c \in C} \pi_c}{|C|} \quad (6)$$

where $|C|$ is the total number of clusters and π_c is the payoff of cluster c defined as

$$\pi_c = \sum_{f \in F^c} \pi^f \quad (7)$$

5.1. Evolutionary stable strategy (ESS)

Evolutionary Stable Strategy is a stability concept that was also proposed by John Maynard Smith for populations of individuals sharing a common behavioral characteristic. ESS was presented for a monomorphic population, where every individual adopts the same strategy. According to [15], ESS makes the following assumptions:

- Players choose their strategies from identical sets.
- The payoff to a player choosing a particular strategy against a competitor choosing another strategy is the same regardless of the characteristics of the players.
- Players cannot condition their choice of strategies based on any characteristics of players.

Consider player f using a strategy S_k and its expected payoff $\pi^f(S_k, \hat{S})$ considering that \hat{S} is the strategy used by another player. Then, ESS for a monomorphic population is defined as

Definition 2 - Evolutionary stable strategy. A strategy S^* is an ESS if and only if for all $S_k \neq S^*$ we have

$$\pi^f(S_k, S^*) \leq \pi^f(S^*, S^*) \quad (8)$$

$$\pi^f(S_k, S_k) < \pi^f(S^*, S_k) \quad \text{if} \quad \pi^f(S_k, S^*) = \pi^f(S^*, S^*) \quad (9)$$

where $\pi^f(S_k, S^*)$ refers to the payoff for the player using strategy S_k . Eq. (8) implies that strategy S^* is the best response to itself. It defines the equilibrium condition while Eq. (9) defines the stability condition. The latter states that if a mutant strategy, S_k , is an alternative best response against the incumbent strategy, S^* , then the average payoff of S^* is higher than the average payoff of S_k .

ESS focuses on a static definition to capture the dynamic process of natural selection. However, models of natural selection are more likely to be dynamic, i.e. based on theories of dynamical systems and stochastic processes. In this sense, Taylor and Jonker [5] defined the replicator dynamic that is the most important game dynamics studied in EGT.

5.2. Replicator dynamics and stability definition

Replicator dynamics studies the dynamic evolutionary games through a differential equation that determines the rate of growth of a specific strategy. An individual from a population is called replicator if it is able to replicate itself through the process of selection. Thus, a replicator with a higher payoff will replicate itself faster. This strategy adaptation process is modeled by using a set of ordinary differential equations called replicator dynamics [26] defined as

$$\dot{x}_c(t+1) = x_c(t)[\pi_c - \bar{\pi}] \quad (10)$$

where $x_c(t) = \frac{|F^c|}{N_f}$ represents the cluster c population share at iteration t , π_c is the payoff of cluster c , and $\bar{\pi}$ is the average femtocell payoff in all clusters.

The replicator dynamics consider the payoff of cluster c , π_c , and the average payoff $\bar{\pi}$ of all clusters. Thus, in order to evaluate the replicator dynamic, each femtocell within cluster c observes its payoff and compares it with the average payoff of all clusters. If its payoff is less than the average payoff of the femtocells in cluster c , it will select a_2 strategy and move out to another cluster. According to the replicator dynamics, the population share or the proportion of femtocells choosing strategy a_2 (leaving the cluster) will increase if their payoff is less than the average payoff. In addition, the replicator dynamics are used to evaluate the cluster stability. Thus, when the replicator dynamics are

Table 3
Components of the proposed k-EGT framework.

Components	Description
Players	Set of femtocells is $F = \{F_1, F_2, \dots, F_f, \dots, N_f\}$.
Set of Strategies	Femtocells will decide either to switch or not to a new cluster depending on the achieved payoff of their current clusters. The set of possible strategies for each femtocell $S = \{S_k\} = \{a_1, a_2\}$ is defined by possible actions, where a_1 and a_2 refers to staying in the femtocell's current cluster and to switching to another cluster, respectively.
Population Share	The set of femtocells constitutes the population in our k-EGT model. Thus, a portion of the femtocells will join a cluster by choosing the a_2 action while the rest of femtocells will remain in their current clusters by choosing a_1 action. Consequently, the population share of cluster c is given by $x_c(t) = \frac{ F^c }{N_f}$, where $ F^c $ represents the number of femtocells within cluster c and N_f is the total number of femtocells.
Payoff function	The payoff of a cluster depends on the throughput achieved for all femtocells within the cluster as explained in Section 5.

equal to zero, $\dot{x}_c(t + 1) = 0$, the clusters' stability is achieved since the payoff of each cluster is similar to the average payoff of all clusters, i.e. $\pi_c = \bar{\pi}$. Consequently, no femtocell will change its strategy and move out of its current cluster since its payoff is equal to the average payoff of all the population.

According to [27], the replicator dynamics give the connection between the dynamic evolutionary equilibrium (EE) and the ESS. Consequently, the ESS of our evolutionary game can be derived by finding the EE of the replicator dynamics. In the replicator dynamics, there exist the boundary EE and the interior EE. The boundary EE is given when a population share $x_{c_k} = 1$ and thus $x_{c_j} = 0$ for all $j \neq k \in S$. On the other hand, the interior EE corresponds to $x_{c_k}^* \in (0, 1), \forall k \in S$. According to our work, $x_{c_k}^*$ is an interior EE of the replicator dynamics. The demonstration is that the payoffs achieved by femtocells within clusters are similar to the average payoff of all clusters. Thus, it is demonstrated that the payoff obtained is strictly higher than the payoff obtained when femtocells decide to keep in the clusters where the payoff is lower than the average payoff. This solution is considered as the Nash equilibrium and since any strict Nash equilibrium corresponds to an ESS [28], it is demonstrated that our approach reaches the ESS.

6. Femtocell clustering based on evolutionary game theory

In this section, the k-EGT framework is presented for the clustering of femtocells in a macro-femtocell network. Initially femtocells are clustered using the K-means algorithm [29]. In addition, the resource allocation is performed within every cluster using a PSO algorithm.

6.1. k-EGT framework

The k-EGT framework consists of an initial formation of clusters using the K-means algorithm, an evolutionary game to balance clusters based on the cluster payoffs and a resource allocation carried out using the PSO algorithm. In Table 3, we describe the components of the proposed k-EGT framework such as the players, the set of strategies, the population share, and the payoff function. Furthermore, the steps to form evolved clusters of femtocells are

- Initial formation of clusters using the K-means algorithm.
- Evolutionary clustering where femtocells choose to switch clusters or not.
- Distributed resource allocation using the PSO algorithm within every evolved and stable cluster.

6.2. k-EGT clustering algorithm

This section describes the femtocell clustering using the k-EGT model that is used in Algorithm 1. The clustering approach is used to reduce the complexity of the resource allocation in a two-tier network. As already assumed in Section 4, the resources are split between macro-tier and femto-tier in order to eliminate the cross-tier interference. Concerning the co-tier interference, it is reduced by clustering since each cluster head optimizes locally the resource allocations.

To group femtocells into clusters, an initial clustering process is made using the K-means algorithm. The first step of K-means algorithm is to arbitrarily select an initial femtocell partition among N_c clusters. This initial partitioning is based on location points chosen randomly within the femtocells area coverage. These location points, also known as centroids, are treated as cluster centers. Then, each femtocell is allocated to the cluster whose centroid is closest to the femtocell. Then the centroid locations are adjusted based on the current allocation of femtocells to clusters and the allocation process is repeated.

The proposed k-EGT model is performed to balance the formed clusters towards stable clusters. To do that, femtocells within clusters with payoffs smaller than the average payoff ($\pi_c < \bar{\pi}$) leave their current clusters and join clusters with payoff larger than the average payoff. This also avoids having overcrowded clusters. The femtocells that leave their current cluster need to choose a cluster from the set of clusters with payoff larger than the average payoff. Consequently, any cluster with $\pi_c > \bar{\pi}$ can be chosen and the selection of a particular cluster is done with probability [14] defined as

$$p_c = \frac{\dot{x}_c - x_c}{\sum (\forall h)(\dot{x}_h - x_h)}; \pi_c > \bar{\pi}, \pi_h > \bar{\pi} \tag{11}$$

Algorithm 1: Evolutionary clustering algorithm.

Input: Initially, clusters are formed using the K-means algorithm. So there are totally N_c clusters.

Output: Set of stable clusters, $\beta_u^{f,s}$, $P_u^{f,s}$, R_{SU}^f , R_{PU}^f

Step 1 - Cluster Head Selection

for each cluster $c \in C$ do

Determine the members of the cluster c .

for each member of the cluster c do

Calculate the number of neighbors.

end

Select the member that has the maximum number of neighbors as cluster head of cluster c .

end

Step 2 - Evolutionary Cluster Formation

for each cluster $c \in C$ do

Compute the payoff of the cluster, π_c , based on the demanded data rate of PUs served by FCs within the cluster.

Compute the average payoff of all the clusters according to Eq. (6).

Evaluate the stability by applying Eq. (10).

Determine the set of stable clusters by verifying that the payoff of every cluster is equal than the average payoff.

end

Step 3: Resource Allocation per Cluster

for each cluster $c \in C$ do

Determine the set of users for the current cluster c .

Run the PSO based resource allocation algorithm for the mobile users in the cluster.

end

6.3. Resource allocation based on particle swarm optimization

According to our model, a specific amount of the macrocell bandwidth is dedicated to the formation of the femtocell clusters. Consequently, the total number of available subcarriers (N_s) is divided into macro-tier subcarriers and femto-tier subcarriers which eliminates the

cross-tier interference. When the clusters are established, the cluster head of every cluster receives information of the corresponding subcarriers for its cluster. Then, the cluster head performs an orthogonal allocation of subcarriers to every femtocell within the cluster based on the PSO algorithm, and this orthogonal allocation reduces the intra-cluster interference.

In our previous work, [4], we demonstrated that a Particle Swarm Optimization (PSO) algorithm gives a satisfying near-optimal solution and speeds up the optimization process. Therefore in our evolutionary approach, the resource allocation within each cluster is based on a PSO based algorithm. PSO has been already used for the resource allocation in OFDMA macrocell systems [30] and in LTE systems [31]. In [32], it was demonstrated that the resource allocation based on PSO requires between 100 to 1000 iterations to converge to a solution. The implementation of PSO requires relatively small number of code lines due to the use of simple operations. In particular, it takes only one operation to update the velocity and position to coordinate and control the particles movements. In this technique, no overlapping and mutation calculations are involved. In addition, PSO demands less time to find solutions when compared to genetic algorithms [33].

The PSO algorithm simulates the social behavior of animals living in swarms [34]. It initializes with a population of particles where each particle stands for a candidate solution to a problem. PSO has three main attributes: the position in the search space l , the current velocity v , and the best position ever found by the particle during the search process. In order to determine the position and velocity of each particle n at each iteration t , PSO algorithm uses two vectors that are updated based on the memory gained by each particle. Thus, the position l_n^{t+1} and velocity v_n^{t+1} of a particle n at each iteration t are updated as follows:

$$l_n^{t+1} = l_n^t + \delta_t v_n^t, \quad (12)$$

$$v_n^{t+1} = \omega v_n^t + d_1 r_1 (p_n^{local} - l_n^t) + d_2 r_2 (p_n^{global} - l_n^t) \quad (13)$$

where δ_t is the time step value typically considered as unity [35], p_n^{local} and p_n^{global} are the best ever position of particle n and the best global position of the entire swarm so far, and r_1 and r_2 represent random numbers from interval [0,1]. Moreover, parameters ω , d_1 and d_2 are the configuration parameters that determine the PSO convergence behavior, the values of these parameters are indicated in Table 4.

The applied PSO algorithm, Algorithm 2, is executed by the cluster head that allocates the bandwidth within each cluster. The selection of the cluster head is based on the maximum number of neighbors that a cluster member has, see Algorithm 1.

6.4. Benchmark models

We compare our model with four benchmark models. The first one, SH-PSO, is a distributed clustering model that was presented in [3]. This model employs the PSO algorithm that is performed locally within each cluster. In this model the resources are allocated in a fair manner since the Shapley value is used. The second benchmark model, named as load balanced clustering (LBC) model, is a centralized model that was presented in [4]. It uses the Weighted Water Filling (WWF) algorithm for the resource allocation. Furthermore, the LBC model proposes a femtocell power control to mitigate interference and to achieve a target SINR. The third one, PSO-Dist, proposes a distributed clustering model based on a cooperative game, where femtocells are encouraged to form clusters while being rewarded with resources from the macrocell [36]. The fourth model named SDN-HAC tackles the femtocell clustering by using a suitable function based on the value of each cluster. In this model, femtocells are considered to work in closed access mode, thus, femtocells only give service to their subscribers [37]. The main difference between the proposed model and the benchmark models is that the proposed model performs an analysis of the cluster stability during the clustering process using EGT. Moreover, the mobility of users in a dense femtocell-deployment scenario is added in this paper.

Algorithm 2: PSO based resource allocation algorithm

Input: *MU Locations* (l_u, y_u), *Set of femtocell members of the cluster* (F^c), *users demands* (D_u), *BS selection per user* (α_u^f), *bandwidth per cluster* (BW_c).

Output: *Bandwidth and power allocation per user* (b_u, P_u).

for each $u \in MU$ **do**

$$b_u^{max} = \frac{D_u}{\gamma_f};$$

$$P_u^{max} = \min(P_u^{max}, SINR_f^{max} \times (\sigma + I_{th}) \times PL_u^f);$$

end

Generate initial swarm with the particle positions and velocities as follows;

$$\mathbf{b} = \mathbf{r}_1 \cdot \mathbf{b}^{max};$$

$$\mathbf{P} = \mathbf{P}^{min} + \mathbf{r}_2 \cdot (\mathbf{P}^{max} - \mathbf{P}^{min});$$

$$\mathbf{v}_b = \mathbf{r}_3 \cdot \mathbf{b}^{max};$$

$$\mathbf{v}_P = \mathbf{P}^{min} + \mathbf{r}_4 \cdot (\mathbf{P}^{max} - \mathbf{P}^{min});$$

Evaluate Fitness Function;

Determine first global best of the swarm;

while $t \leq MaxIteration$ **do**

Update Position;

Evaluate Fitness Function;

Determine best local for each particle;

Determine best global in the swarm and update the best global;

Update velocity;

end

7. Simulation results

In this section, we present and analyze results of MATLAB simulations that were performed to evaluate the proposed evolutionary game theoretic approach. In particular, we show the performance of the model in terms of subscribers' satisfaction, network throughput, interference, and running times for the clustering process. Our results were compared with the two benchmark models described in Section 6.4.

In the simulations, we consider two scenarios. The first scenario is a non-dense femtocell network and the second scenario considers the increase of the femtocells number to achieve dense-deployment of femtocells in the network. In the first scenario, the number of PUs varies from 10 to 60 with increments of five users. In this case, 10 femtocells are deployed in an area of 500×500 m. In the second scenario, the number of femtocells increases from 10 to 90 while the number of public users remain fixed. For example, for 10 and 90 femtocells the number of public users is 30 and 270, respectively, considering that the maximum number that a femtocell can grant service is 3. In both scenarios, one subscriber is assigned to each FC with variable demand ranging from 128 Kbps to 1 Mbps. Additionally, a dedicated number of macrocell subcarriers is used for the PUs served by femtocells in clusters. This number is defined as $BW_c = b \times B_s \times N_s$, where b is a value between [0, 1] that represents the portion of available subcarriers used by the femto-tier. Besides the case without mobility, a mobility case, with random velocity of 0–4 m/s for the public users is also considered for both scenarios. Table 4 presents the system parameters for the network configuration and the PSO parameters. The clustering process starts with an initial formation of clusters using the K-means algorithm. In order to form the clusters, the K-means algorithm needs to know the number of clusters to form which is set by giving a value to N_c . To find the initial number of clusters N_c , we apply the Average Silhouette Method. This method measures the quality of a clustering by determining the average distance between clusters (the average silhouette width). Thus, a high average silhouette width indicates a good clustering [38]. According to the silhouette method the suggested initial number of clusters is 5, as can be seen in Fig. 2.

The following step in the clustering process is to apply the k-EGT model to the already formed clusters. This k-EGT model is also applied

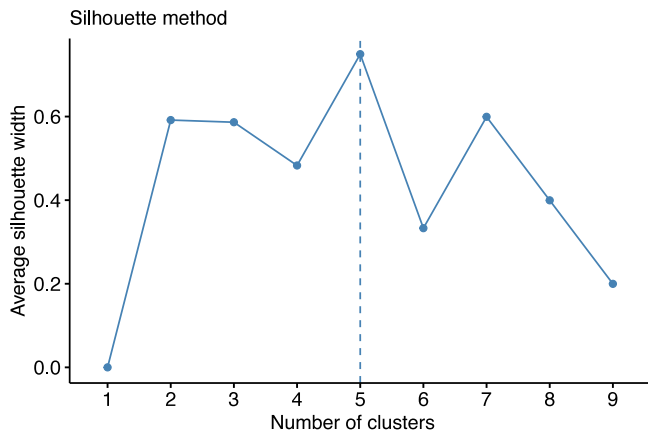


Fig. 2. Optimal number of clusters using the Average Silhouette Method.

Table 4
System and PSO parameter settings.

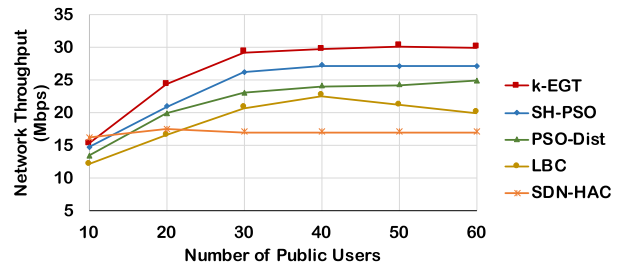
Network Configuration		
Name	Description	Value
N_s	Number of Subcarriers	256
P_{MC}^{Total}	Transmitted power per MC	60 dBm
P_f^{Total}	Transmitted power per FC f	10 dBm
r_{MC}, r_f	Macrocells and femtocell radius	500 m, 20 m
γ_f	Spectral efficiency for FC f	6
W	Wall loss penetration	-3 dB
f_c	Carrier frequency	2300 MHz
σ	Noise	-174 dBm/Hz
$ SU $	Number of subscribers per FC f	1
$ PU $	Number of public users	5–60
N_f	Number of deployed femtocells	10–90
$v(t)$	Users velocity	0–4 m/s
PSO Parameters		
Name	Description	Value
d_1	Cognitive knowledge parameter	2.0
d_2	Social interactions parameter	1.5
ω	Inertia	0.85

every time the number of PUs increases in the case of the first scenario. In Table 5, the entries with times different from zero indicate for which PUs increases there was a need to calculate new cluster formations.

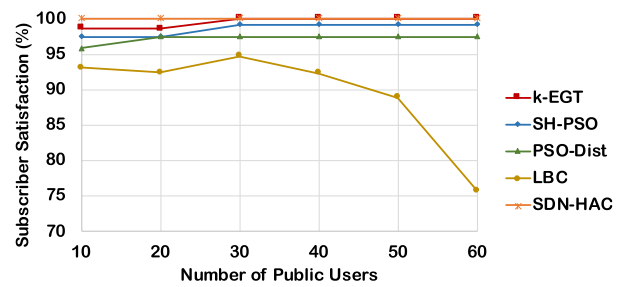
7.1. Scenario without mobility

In this section, we analyze the effect of the evolutionary game theory model (k-EGT) on the network throughput, subscribers' satisfaction, and interference for users without mobility. Fig. 3(a) shows the network throughput with the increasing number of public users from 10 to 60 without mobility. As stated before in Section 6.2, an initial set of clusters is formed using the K-means algorithm. The number of the initial set of clusters to be formed is defined as $N_c = 5$. Thus, the k-EGT model analyzes the 5 clusters formed with 10 femtocells based on the replicator dynamic. When the number of femtocells increases to 20, there is a new formation of clusters that takes 0.0156 s as can be seen in Table 5.

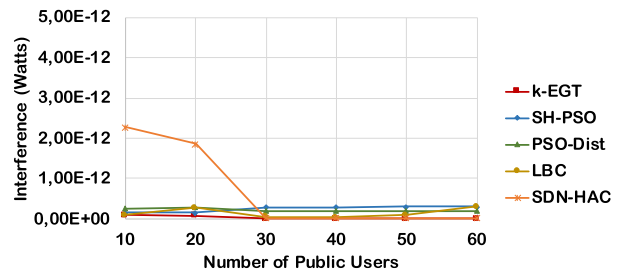
The figure demonstrates that the proposed evolutionary game provides higher throughput than the benchmark models (the centralized LBC model and the distributed SH-PSO, PSO-Dist, and SDN-HAC models). In particular, the throughput gain of the proposed model ranges from 25% to 50% when compared to the LBC model, from 3% to 17% when compared to the SH-PSO model, and from 13% to 27% when compared to the PSO-Dist model. It can also be observed that the lowest throughput is achieved with the SDN-HAC model. This is due to that this model considers femtocells in closed access mode and therefore



(a) Network throughput for variable number of public users without mobility.



(b) Subscribers Satisfaction for variable number of public users without mobility.



(c) Average interference by subcarrier for variable number of public users without mobility.

Fig. 3. Performance metrics vs. number of public users for the case without mobility.

the public users far from the MBS and close to clusters of femtocells cannot be served by femtocells. Consequently, the network throughput is reduced since several public users will not be allocated subcarriers and will be blocked.

We define the satisfaction of subscribers as the ratio of the allocated data rate to the demanded data rate of every user. From Fig. 3(b), it can be seen that our k-EGT model provides the users with higher satisfaction in comparison with the SH-PSO, PSO-Dist and LBC models. Moreover, from 30 PUs, the 100% satisfaction is obtained with the k-EGT model. Furthermore, it is shown that the subscriber satisfaction obtained with the k-EGT model has a gain up to 32% when compared with the LBC model. The subscribers' satisfaction using the SDN-HAC model is 100% from 10 to 60 PUs. This is a consequence of femtocells working in closed access mode and thus the resources allocated to the femtocells are only assigned to their subscribers. From Fig. 3(c), we can conclude that the k-EGT model reduces the interference when compared with the interference generated with the SH-PSO, PSO-Dist, SDN-HAC, and LBC models. In particular, starting from 40 PUs, the interference is zero with the k-EGT model.

In Fig. 4, we show the network throughput vs. number of clusters N_c in the non-dense scenario. As can be seen, the highest network throughput is achieved when N_c takes values from 2 to 5.

Table 5
Running times for the clustering component in a scenario with and without mobility varying the number of PUs from 10 to 60.

No. PUs	Clustering time with mobility (sec)				Clustering time without mobility (sec)			
	SH-PSO	k-EGT	PSO-Dist	SDN-HAC	SH-PSO	k-EGT	PSO-Dist	SDN-HAC
10	1.2031	0.1094	1.1970	0.1399	0.241	0.0625	1.81	0.0999
20	0	0.0156	1.7580	0.0799	0	0.0156	2.22	0.0200
30	0	0	2.0430	0	0.075	0	1.62	0.0399
40	0	0	0	0	0	0	0	0
50	0.375	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0

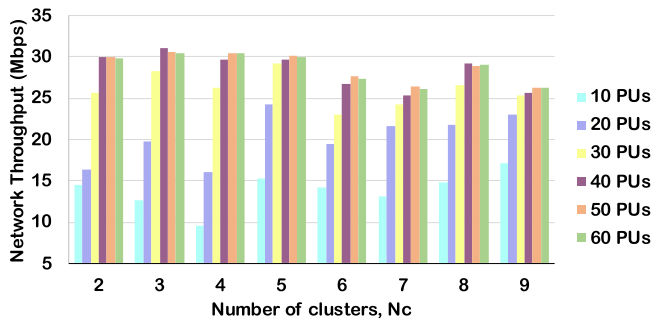


Fig. 4. Network throughput vs. number of clusters N_c for the non-dense scenario.

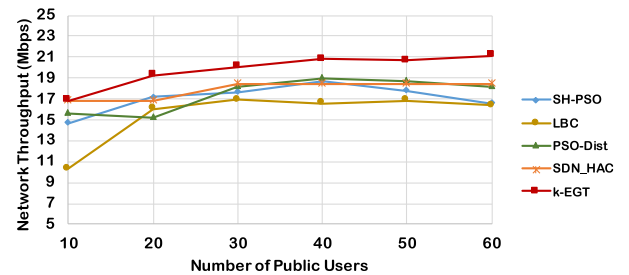
7.2. Scenario with mobility

In Fig. 5(a), we present the network throughput for the scenario with user mobility. As in the case without mobility, the k-EGT model provides higher throughput than the LBC, PSO-Dist, SDN-HAC, and SH-PSO models. Nevertheless, when we compare the network throughput for scenarios with and without mobility, we can observe that higher throughput is obtained when users are static. This is due to the fact that mobile users with higher velocity move out of the nearby femtocell coverage and try to connect to the macrocell.

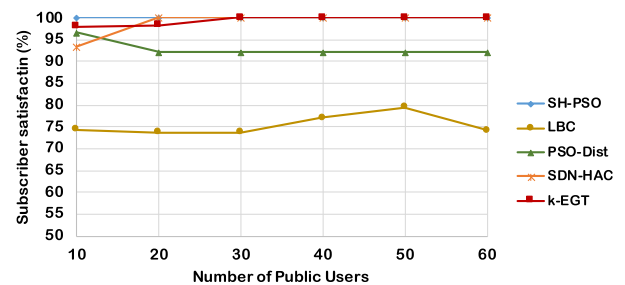
Fig. 5(b) shows that the subscriber satisfaction for the k-EGT and SH-PSO models is similar and is not affected negatively by the users' mobility. For the cases of 10 and 20 PUs, the satisfaction with the SH-PSO model is slightly better than with the k-EGT model due to the fact that in the SH-PSO model subscribers are rewarded with extra-resources. However, the running times for the clustering process of the SH-PSO model are higher than the ones obtained with the k-EGT model, see Section 7.5. With regard to the LBC and PSO-Dist models, the k-EGT model provides gain in the range of 35% and 9% of the subscriber satisfaction, respectively. With the SDN-HAC model the subscriber satisfaction is 100% from 20 PUs which is a consequence of femtocells working in closed access mode. When mobility is added to public users, the interference per subcarrier achieved with our k-EGT model is similar to the interference with the SH-PSO model, see Fig. 5(c). On the other hand, when compared with the LBC model, the interference generated with the k-EGT model is higher. The main reason for this result is that the LBC model applies a power control to mitigate the interference.

7.3. Femtocell dense-deployment

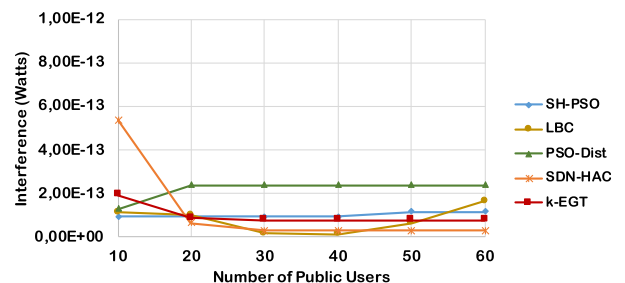
In this section, the network throughput, subscribers' satisfaction, and interference are evaluated for scenarios with and without mobility under a dense-deployment of femtocells. In this case, the k-EGT model is only compared with the centralized LBC model and the PSO-Dist and SDN-HAC models. This is due to the complexity and memory requirements of the SH-PSO model that is prohibitive for Matlab implementation when the number of femtocells is large. The considered



(a) Network throughput using random velocity from 0 m/s to 4 m/s.



(b) Subscribers satisfaction using random velocity from 0 m/s to 4 m/s.

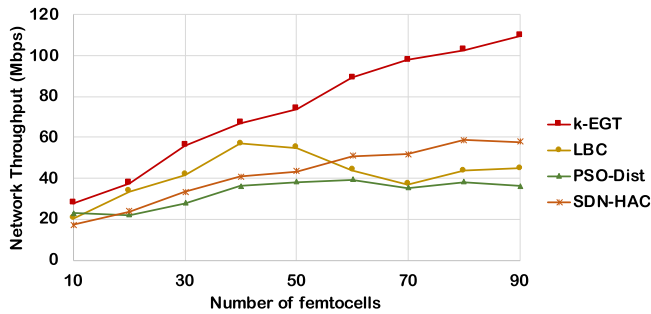


(c) Average interference per subcarrier using random velocity from 0 m/s to 4 m/s.

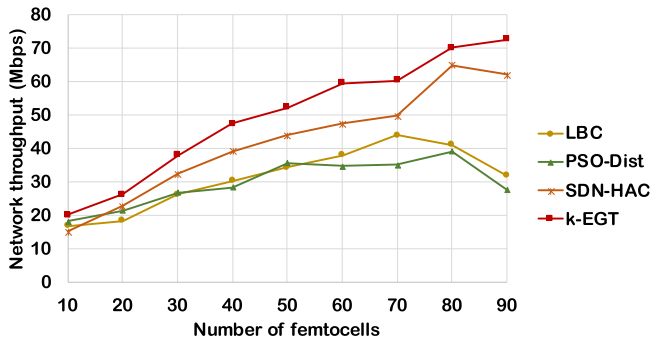
Fig. 5. Performance metrics vs. the number of public users for the case with mobility.

metrics are evaluated by increasing the number of femtocells from 10 to 90. In this scenario, the number of PUs is fixed according to the maximum number of public users that a femtocell can serve, i.e. 3 PUs per femtocell. For example, for 10 and 90 femtocells the fixed number of PUs is 30 and 270, respectively.

For the considered dense-deployment of femtocells, with and without mobility, we conclude that our k-EGT model outperforms the LBC, SDN-HAC, and PSO-Dist models according to the following results. Fig. 6(a) shows that the network throughput of the k-EGT model without mobility is three times higher and four times higher than the throughput obtained with the LBC model and the PSO-Dist model, respectively, while for the case with mobility the k-EGT model gives



(a) Without mobility



(b) With mobility

Fig. 6. Network throughput for the femtocell dense-deployment scenario.

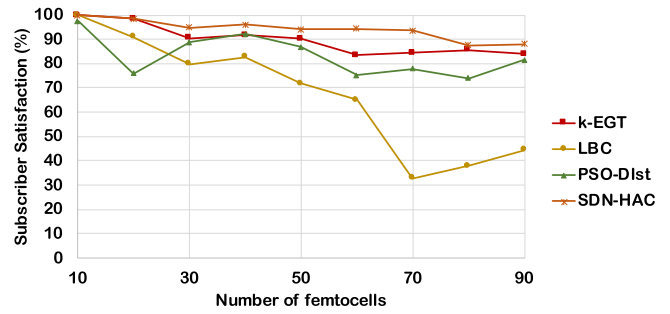
the network throughput three times higher compared with the LBC and PSO-Dist model, as can be seen in Fig. 6(b). When compared the k-EGT model with the SDN-HAC model, our model obtains a throughput gain in the range from 8% to 32% and from 63% to 90%, for the scenarios with and without mobility, respectively.

Regarding the subscribers' satisfaction, Figs. 7(a) and 7(b) show that users achieve higher satisfaction with the k-EGT model than with the LBC and PSO-Dist models. However, it can be observed that the SDN-HAC model achieves higher subscribers satisfaction than our model k-EGT. The reason is that femtocells work in closed access mode in the SDN-HAC model, and thus the resources for the subscribers are guaranteed since they do not have to share them with public users. In addition, it can be observed that satisfaction decreases with an increasing number of femtocells. This is because the throughput is affected by the interference which gets severe when the density of femtocells grows. However, from Figs. 8(a) and 8(b), we conclude that our k-EGT model reduces the interference level below the values obtained with the LBC and the PSO-Dist models, for the mobility and no mobility scenarios.

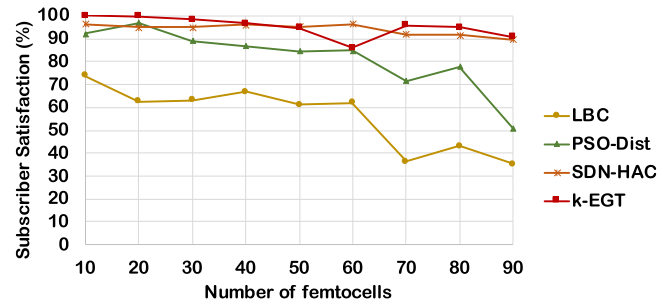
The main reasons for the better performance of our model against the centralized model are that the k-EGT model uses the replicator dynamics to guarantee the stability of the clusters and that the resources allocated to each member of the cluster are fairly allocated since their payoff is similar to the average payoff of all clusters. On the other hand, the LBC model forms balanced clusters that tend to have the same size and that does not guarantee fairness. Furthermore, the LBC and the PSO-Dist models do not consider any criteria to evaluate the stability of the clusters. Regarding the SDN-HAC model, the co-tier interference is highly reduced since this model allows more femtocells to join into clusters, thus forming larger clusters.

7.4. Stability analysis

In the proposed model, stability is obtained by keeping femtocells in their clusters as long as their payoffs are higher or equal than the

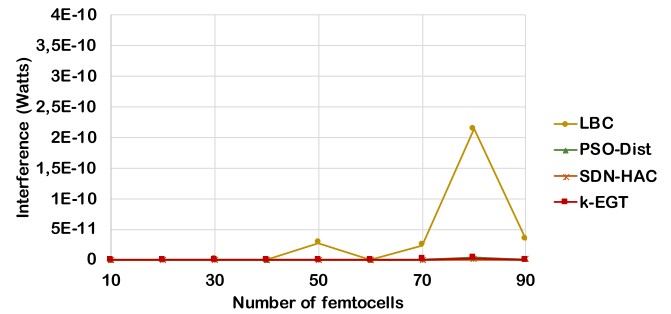


(a) Without mobility

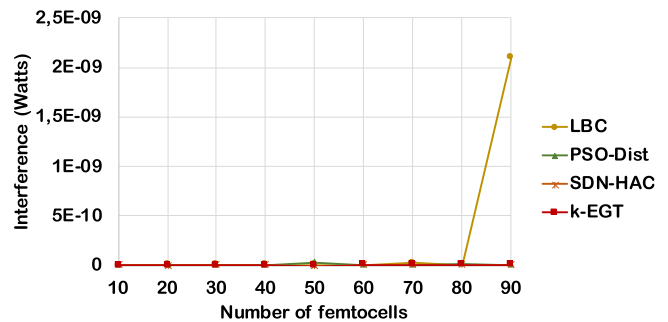


(b) With mobility

Fig. 7. Subscriber satisfaction for the femtocell dense-deployment scenario.



(a) Without mobility



(b) With mobility

Fig. 8. Average interference per subcarrier for the femtocell dense-deployment scenario.

average payoff of all clusters. This stability criteria is based on the replicator dynamic of the evolutionary game theory. In particular, the replicator dynamic states that a cluster is stable if all the clusters have an equal or similar payoff to the average payoff, i.e. $\pi_c = \bar{\pi}$ for all $c \in C$. In Fig. 9 we illustrate the stability of the formed clusters by showing the payoffs obtained by clusters with the k-EGT and LBC models for the case of ten femtocells. The figure shows that the set of clusters formed

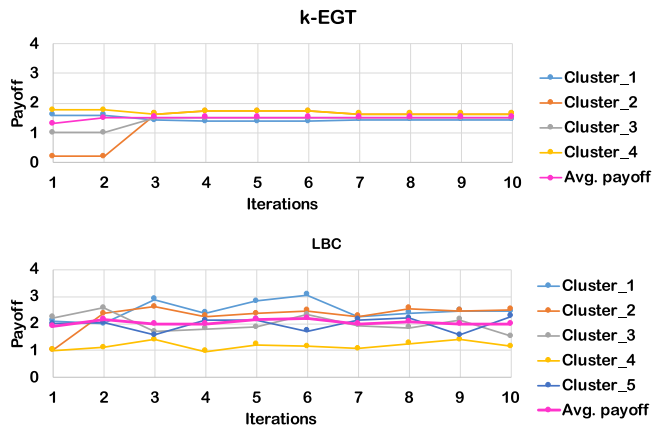


Fig. 9. Average payoff per cluster for 10 femtocells using the k-EGT (k=5) and LBC models.

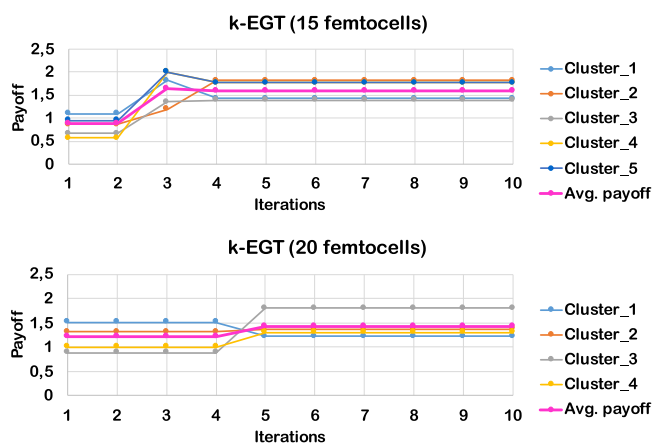


Fig. 10. Average payoff per cluster for 15 and 20 femtocells using the k-EGT model.

with the k-EGT model achieved its stability from iteration three. On the other hand, the clusters of the LBC model do not achieve stability, e.g. the payoff of the fourth cluster is below the average payoff. The results presented in Fig. 10 illustrate that the stability convergence depends on the number of femtocells. In this particular cases with 15 and 20 femtocells, the stability is achieved at iterations four and five, respectively.

7.5. Complexity and running times

The complexity of our solution, forming clusters by the evolutionary game and resource allocation based on PSO algorithm, is $O(f^2)$, where f represents the number of femtocells. Therefore the proposed k-EGT model has an acceptable complexity when compared with the complexity of the exhaustive search $O(f^f)$ [39]. In order to have a more detailed comparison of the complexity of the different algorithms presented in our work in the following the running times are discussed.

We present the running times of the clustering process for scenarios with and without mobility. Table 5 reports the computation time associated with the clustering process of the k-EGT, SH-PSO, SDN-HAC, and PSO-Dist models for different public users density with and without mobility. The first column represents the number of PUs, the second and sixth columns correspond to the clustering time using the SH-PSO model with and without mobility, the third and seventh columns show the clustering time of the k-EGT model with and without mobility, the fourth and eighth columns represent the clustering time of the PSO-Dist model with and without mobility, and the fifth and ninth columns show the clustering times of the SDN-HAC model, respectively.

Table 6
Clustering times using K-means and k-EGT algorithms.

No. FCs	Nc	K-means	k-EGT
10	5	0.02	0.09375
20	5	0.02	0.09375
30	5	0.02	0.07813
40	5	0.03	0.07813
50	5	0.03	0.07813
60	10	0.03	0.07813
70	10	0.03	0.17188
80	10	0.03	0.14063
90	10	0.03	0.14063

Note that in the scenario without mobility the running times are significant only for the cases with 10 and 30 PUs for the SH-PSO model. This is owing to the fact that only in these cases there is formation of new clusters. On the other hand, the evolutionary solution stops forming clusters from 30 PUs since stability was found at that moment. This means, that all the clusters are stable and the achieved payoff is equal to the average payoff.

For the mobility scenario and the k-EGT model, the formation of clusters stops from 30 PUs similarly to the no mobility scenario this means that stability was achieved. On the other hand, the SH-PSO model stops forming clusters for 20, 30, 40 and 60 PUs when stability is found. In this scenario, it can be observed that for the initial set of clusters with 10 PUs, the clustering process takes 1.2031 s for the SH-PSO model while the k-EGT model takes a much lower time of 0.1094 s.

It can be observed that the higher clustering times are obtained with the PSO-Dist model for both scenarios with and without mobility. In particular, after 30 PUs the running time of the clustering process becomes 0 meaning that neither the clusters can increase their utility by admitting new femtocells nor the femtocells can get extra resources to increase their subscribers' satisfaction. On the other hand, the clustering times achieved with the SDN-HAC model are slightly larger when compared with the k-EGT model. In particular, the clustering process ends from 30 PUs and from 40 PUs, for the mobility and no mobility scenarios, respectively. This means, that from 30 and 40 PUs the suitability function used to form the clusters is negative or that the grand cluster was formed.

In Table 6, the running times for the clustering process for different FC numbers are presented. It can be observed that the clustering times achieved by the k-EGT model are low. It is worth noting that the initial set of clusters is formed by using the K-means algorithm, e.g. we set the number of clusters to 5 ($N_c=5$) for a number of 10 to 50 FCs and to 10 ($N_c=10$) from 60 to 90 femtocells, as can be seen in the second column of Table 6. Consequently, the cluster size is not large, thus the resource allocation solved by the cluster head within each cluster converges within a short time.

8. Conclusion

In this paper, we addressed the problem of resource allocation for dense femtocell networks by proposing a model that forms stable clusters using an evolutionary game where femtocells learn from the environment and make their decisions considering the achieved payoff related to the throughput. In order to guarantee the cluster stability, we use the replicator dynamics that find the evolutionary equilibrium of the evolutionary game. In particular, we show that the stability is achieved when the payoff of each cluster is equal to the average payoff of all clusters. In addition, Particle Swarm Optimization is used for the local resource allocation within each cluster since this approach provides near-optimal solution while speeding up the optimization process. Two scenarios were analyzed by means of simulations, the

first one having a variable number of public users and the second one with increasing number of femtocells. For the non-dense femtocell deployment (10 femtocells in the considered scenario) the results show that the network throughput improves significantly (up to 50%) when compared with the centralized LBC model and up to 27% when compared with the PSO-Dist model. The improvement is smaller (up to 17%) when compared with the SH-PSO model, which is due to the fact that the SH-PSO model has a fair allocation of resources by means of the Shapley value and this is not present in the LBC model. While the SH-PSO model provides better throughput than the LBC model, its complexity is significant and even prohibitive for the considered dense femtocell deployment (90 femtocells in the considered scenario). In high-density scenario, the comparison between the k-EGT and LBC models indicated that the throughput of our model can be increased by factor of three for the static mobiles case and by factor of three for the case with user mobility, respectively. When compared with the PSO-Dist model, the k-EGT model increases the throughput by factor of four for the static mobiles case while for the case with user mobility the throughput is increased by factor of three.

CRedit authorship contribution statement

Katty Rohoden: Methodology, Software, Validation, Writing - original draft. **Rebeca Estrada:** Validation, Supervision. **Hadi Otrók:** Conceptualization, Supervision. **Zbigniew Dziong:** Conceptualization, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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